EFFECT OF CONSTANT AND UNIFORM HEAT GENERATION ON THE THERMAL BEHAVIOUR OF POROUS SOLIDS WITH ASYMMETRIC BOUNDARY CONDITIONS

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Abstract. The generation of heat due to chemical reaction will have a significant effect on the temperature profile and heat transfer within a porous body. Most forms of analysis only consider the symmetric situation or else make use of various assumptions that greatly simplify the analysis, for example: the Semenov or the Frank-Kamenetskii models. The objective of this paper is to develop an improved understanding of the thermal behaviour of a porous body with uniform internal heat generation, which is in contact with two fluids at different temperatures and with different heat transfer coefficients. The mathematical representation is a one dimensional Poisson equation with asymmetric boundary conditions. The analytical solution reveals four regimes for heat flow: (a) purely conduction at zero heat generation, (b) a combination of heat flow by conduction through the body between the hot and cold fluids and all heat generated passing to the colder fluid, (c) no heat flow by conduction between the two fluids and all heat generated passing the cold flow - the so-called critical heat generation, and (d) the heat generated passes to both the cold and hot fluids and there is a maximum temperature within the body greater than that of the hot fluid, the so-called supercritical region. Expressions are developed to allow predictions of the conditions pertaining to each regime. This new representation covers the Semenov and Frank-Kamenetskii models and all possible solutions intermediate of the them.

Keywords. Frank-Kamenetskii, Semenov, asymmetry, critical heat generation, maximum temperature.

1. Introduction

Many porous solids have internal heat generation, whether it is caused by electrical resistance heating, micro-wave heating, exothermal chemical reaction or nuclear fission. These heat generation processes are all temperature dependent, which leads to non-linear differential equations. In all situations the heat generated within the porous body must be removed from the surface of the body, otherwise the temperature of the body will increase dramatically until it melts or in the case of chemical reactions, there will a thermal runaway. There is the potential of severe explosions and the onset of fires.

Most forms of analysis only consider the symmetric situation, so that the geometry of the body may be taken to be one-dimensional and the temperature dependent heat release is linearised, so that an analytical solution can be investigated. In addition, further simplifications are introduced such as the Semenov model, where the thermal conductivity of the porous body is taken to be infinite, or the Frank-Kamenetskii model, where the heat transfer from the surface of the porous body is infinite. In the Semenov model, the temperature throughout the porous body is the same, whereas in the Frank-Kamenetskii model, the temperature of the surface of the porous body is the same as the surroundings. For both of these models, the temperature within the porous body will always be greater than that of the surroundings and so only two conditions will prevail: a stable state and an unstable state. In the stable state, the heat released is dissipated to the surroundings without excessive temperatures existing within the porous body. However, in the unstable state, the heat released within the body cannot be adequately dissipated to the surroundings and this leads to excessive temperatures within the porous body – the critical condition. In the case of chemical reactions occurring within porous bodies, the critical condition is defined by the Critical Ambient Temperature (CAT). This value is determined from experimental investigations and the value decreases as the size of the porous body becomes larger and the rate of heat transfer from the surface of the body into the surroundings becomes smaller. The prediction of stable conditions for a porous body is obtained from the evaluation of the CAT (Babrauskas 2003).

For asymmetric situations, the prediction of a suitable value of the CAT is not possible as yet, although very crude approximations have been suggested (Babrauskas 2003). Furthermore, detailed investigations of the thermal behaviour of a porous solid body subjected to asymmetric boundary conditions and with internal heat generation deserve further investigation. The objective of this presentation is to develop an improved understanding of the asymmetric situation. This will be achieved by assuming, the heat release to be uniform throughout the porous solid body and independent of temperature, which will facilitate an analytical solution of a mathematical representation of the system. Many heat transfer text books consider uniform heat generation in solid objects, and Incropera and DeWitt 2002 provides an
excellent introduction to the topic. However, the presentation does not provide analytical solutions that involve the flux boundary conditions, but suggest how the simpler model of fixed surface temperatures (the Frank-Kamenetskii model) can be adjusted to accommodate these more realistic boundary conditions. The analytical solutions for both the symmetric and asymmetric mathematical representations, which incorporate the flux boundary conditions will be investigated in this paper.

2. Mathematical Models

The temperature, \( T \), distribution within the porous solid body will be taken to be one dimensional in the direction \( n \) normal to the surface of the body and the heat released per unit volume is denoted by \( \dot{W}_f \). The porous solid has a thermal conductivity, \( \lambda \), which is constant throughout the body and independent of temperature. The following Poisson ordinary differential equation must be solved to find the temperature distribution within the porous solid body:

\[
\frac{1}{n^{(i-1)}} \frac{d}{dn} \left( n^{(i-1)} \frac{dT}{dn} \right) + \frac{\dot{W}_f}{\lambda} = 0
\]  

(1)

This Eq. (1) must be solved in the following domain of interest, \( N_2 \leq n \leq N_1 \), where for planar coordinates \( i=1, N_2=0, N_1=L/2 \) and \( n=x \) (thickness \( L \) is very small in relation to the other dimensions), for a very long solid cylinder of diameter \( D \), \( i=2, N_2=0, N_1=D/2 \) and \( n=r \), and for a solid sphere of diameter \( D \), \( i=3, N_2=0, N_1=D/2 \) and \( n=r \).

2.1. Symmetrical Porous Solid Bodies With Constant Uniform Heat Generation

The Eq. (1) is solved with the following boundary conditions for the symmetric situation:

at \( n=N_2 \), \( \frac{dT}{dn} = 0 \)  

(2)

and at \( n=N_1 \), \( -\lambda \frac{dT}{dn} = \alpha(T - T_0) \)  

(3)

where \( \alpha \) is the convective heat transfer coefficient at the surface of the solid porous body and \( T_0 \) is the temperature of the surroundings.

Defining a dimensionless temperature and distance, and introducing the definition of the Biot number, \( Bi \) as follows:

\[
\phi = \frac{(T - T_0)}{\dot{W}_f N_1^2 / \lambda}, \quad \eta = \frac{n}{N_1}, \quad Bi = \frac{\alpha N_1}{\lambda}
\]  

(4)

The Eq. (1) and the boundary conditions (2) and (3) become

\[
\frac{1}{\eta^{(i-1)}} \frac{d}{d\eta} \left( \eta^{(i-1)} \frac{d\phi}{d\eta} \right) + 1 = 0
\]

(5)

\[
\eta=0, \quad \frac{d\phi}{d\eta} = 0
\]

(6)

\[
\eta=1, \quad -\frac{d\phi}{d\eta} = Bi\phi
\]

(7)

The solution of Eq. (5), (6) and (7) provides the dimensionless temperature profile and is

\[
\phi = -\frac{\eta^2}{2i} + \frac{1}{i} \left( \frac{1}{2} + \frac{1}{Bi} \right)
\]

(8)

The centre line dimensionless temperature, which is the maximum value in the solid porous body is

\[
\phi_{\eta=0} = \frac{1}{i} \left( \frac{1}{2} + \frac{1}{Bi} \right)
\]

(9)

and the surface dimensionless temperature is
The temperature difference between the centre line of the object and the temperature of the surroundings is obtained from Eq. (9) by substituting in the definitions of the dimensionless temperature and distance, and the Bi number from Eq. (4):

$$ \phi_{n=1} = \frac{1}{i \text{Bi}} $$

(10)

In Eq. (11), $N_i/i$, is the ratio of the volume $V$ to the surface area $A_s$ of the solid porous body and the heat released within the body is $\dot{Q} = W_i V$, so Eq. (11) can be rearranged as follows:

$$ T_{n=0} - T_w = \frac{W_i N_i}{i} \left( \frac{N_i}{2 \alpha} + \frac{1}{\alpha N_i} \right) \equiv \frac{\dot{Q}}{2 \alpha A_s} \left( \frac{N_i}{2 \alpha A_s} + \frac{1}{\alpha A_s} \right) $$

(11)

Hence the maximum temperature within the solid porous body depends upon the temperature of the surrounding fluid, the total heat generated, the size and the geometry of the body, the thermal conductivity of the porous material and the heat transfer coefficient at the surface. The equations pertaining to the Semenov and Frank-Kamenetskii models are obtained from Eq. (12).

The Semenov model (Centre for Chemical Process Safety 1995) assumes the temperature within the solid porous body to be constant, which is equivalent to the thermal conductivity having a value of infinity, so Eq. (12) becomes

$$ T_{n=0} - T_w = \frac{W_i N_i}{i \alpha} = \frac{\dot{Q}}{\alpha A_s} $$

(13)

The Frank-Kamenetskii (Babrauskas 2003) model assumes the heat transfer coefficient to be infinite, so that the temperature of the surface of the body is that of the surrounding fluid, so Eq. (12) becomes

$$ T_{n=0} - T_w = \frac{W_i N_i^2}{2 i \lambda} = \frac{\dot{Q} N_i}{2 \lambda A_s} $$

(14)

2.2. Asymmetric Porous Solid Bodies With Constant Uniform Heat Generation

The Eq. (1) is still applicable for the asymmetric porous solid bodies, but the dimensions of the three geometries change. The domain of interest is still, $N_2 \leq n \leq N_1$, but now for planar coordinates $i=1$, $N_2=-L$, $N_1=L$ and $n=x$ (thickness $2L$ is very small in relation to the other dimensions), for a very long hollow cylinder of outer diameter $D=2r_2$, $i=2$, $N_2=r_1$, $N_1=r_2$ and $n=r$, and for a hollow sphere of outer diameter $D=2r_2$, $i=3$, $N_2=r_1$, $N_1=r_2$ and $n=r$.

The boundary conditions are now

at $n=N_2$, $-\lambda \frac{dT}{dn} = \alpha_z (T_{n=2} - T)$

(15)

and at $n=N_1$, $-\lambda \frac{dT}{dn} = \alpha_i (T - T_{n=1})$

(16)

where the subscripts on the heat transfer coefficients and the surrounding temperatures refer to positions 1 and 2 respectively.

The Eq. (1) with the boundary conditions (15) and (16) have not been solved for the general situation, but only for a very large slab geometry of finite width and for a very long hollow cylinder. The details of the investigation of the very slab geometry will used to illustrate the thermal behaviour of the asymmetrical situation. For other situations such as one flux boundary condition and an adiabatic boundary condition the reader is referred to the solutions in the book by Incropera and DeWitt 2002.
2.3. Asymmetric Infinite Sized Porous Slab of Finite Width and Constant Uniform Heat Generation

Figure 1 is a schematic of the asymmetric situation under consideration.

Figure 1. Schematic diagram of the asymmetric heating of a large slab with a uniform internal heat source of $\dot{W}_c$.

Eq. (1) in planar coordinates is

$$\frac{d^2 T}{dx^2} + \frac{\dot{W}_c}{\lambda} = 0$$

(17)

and the boundary conditions (15) and (16) become

at $x = -L$, $-\lambda \frac{dT}{dx} = \alpha_2 (T_{\infty} - T)$

(18)

and at $x = +L$, $-\lambda \frac{dT}{dx} = \alpha_1 (T - T_{\infty})$

(19)

The temperature profile in the solid porous slab is obtained by integrating Eq. (17) twice with respect to $x$ and is given by the following equation:

$$T = \frac{\dot{W}_c}{2\lambda} x^2 + C_1 x + C_2$$

(20)

where $C_1$ and $C_2$ are constants of integration. Expressions for $C_1$ and $C_2$ are determined from the boundary conditions (18) and (19), and are found to be:

$$C_1 = \frac{(T_{\infty} - T_{\infty}) + \dot{W}_c L \left( \frac{1}{\alpha_1} - \frac{1}{\alpha_2} \right)}{2L + \frac{\lambda}{\alpha_1} + \frac{\lambda}{\alpha_2}}$$

(21)

$$C_2 = \frac{\left( T_{\infty} + \dot{W}_c \frac{L^2}{2\lambda} + \frac{\dot{W}_c}{\alpha_2} L \left( L + \frac{\lambda}{\alpha_1} \right) \right) + \left( T_{\infty} + \dot{W}_c \frac{L^2}{2\lambda} + \frac{\dot{W}_c}{\alpha_1} L \left( L + \frac{\lambda}{\alpha_2} \right) \right)}{2L + \frac{\lambda}{\alpha_1} + \frac{\lambda}{\alpha_2}}$$

(22)
It is interesting to note that in both the above expressions, the denominator is the reciprocal of the overall heat transfer coefficient $U$ normally employed to calculate the heat flow across a wall from the source to a sink by convection and conduction. If the surface area of the wall is $A$, then the heat flow is obtained from the following relationship:

$$ Q = U A \left( T_{w2} - T_{w1} \right) $$

where the reciprocal of $U$ is given by the expression:

$$ \frac{1}{U} = \frac{2L}{\lambda} + \frac{1}{\alpha_1} + \frac{1}{\alpha_2} $$

### 2.4. Temperature Profiles in an Asymmetric Infinite Porous Slab With Constant Uniform Heat Generation

The temperature profile within the porous solid slab depends upon all the parameters of the problem involved in Eq. (20), (21) and (22). If all the parameters remain constant, except for the value of the heat generation per unit volume $\dot{W}_v$, the shape of the temperature profile and location of the highest temperature within the porous solid slab is determined by this value. Three temperature profiles are displayed in Fig. 2. There are two distinct regions on the temperature/distance plot: **region I**, where the highest temperature in the porous slab is below the hottest temperature of the surroundings and **region II**, where a maximum temperature occurs within the porous slab and this is always greater than the largest temperature of the surroundings. The maximum temperature occurs when the gradient of the temperature is zero:

$$ \frac{dT}{dx} = 0 \Rightarrow -\dot{W}_v \frac{1}{\lambda} x + C_1 $$

The position of the maximum temperature is

$$ x = -L \frac{\frac{1}{\alpha_1} - \frac{1}{\alpha_2}}{\frac{2L}{\lambda} + \frac{1}{\alpha_1} + \frac{1}{\alpha_2}} \left( T_{w1} - T_{w2} \right) $$

The maximum temperature is obtained by substituting Eq. (26) into Eq. (20).

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**Figure 1**: Temperature profiles for varying values of $\dot{W}_v$.

**Region I** exists for values of the heat generation bounded as follows: $0 \leq \dot{W}_v \leq \dot{W}_{v,\text{critical}}$. For $\dot{W}_v = 0$, there is no heat generation and the temperature profile is a straight line and the flow of heat is given by Eq. (23). The heat flow between $T_{w1}$ and $T_{w2}$ is by conduction across the porous slab and by convection at the surfaces of the porous slab. The governing differential equation, Eq. (17) simplifies to the Laplace equation for this condition. As $\dot{W}_v$ increases, heat flow between $T_{w1}$ and $T_{w2}$ still occurs across the slab, but to a lesser extent. The temperature at the hot boundary
approaches that of the hot fluid, thereby reducing the driving force. There is a critical value of the rate of heat generation \( W_{c,\text{critical}} \), when the temperature of the surface at the hot side becomes equal to that of the hot surroundings and the value of the temperature gradient is now zero at the hot boundary of the slab. The value of this critical value of the heat generation in general terms of the hot and cold surrounding temperatures is obtained from Eq. (26) and is given by the following expression:

\[
W_{c,\text{critical}} = \frac{T_{\text{hot}} - T_{\text{cold}}}{\frac{2L^2}{\lambda} + \frac{2L}{\alpha_{\text{cold}}}}
\]  

(27)

There is now no heat flow between the hot and cold surrounding temperatures and all the heat generated within the porous solid slab passes to the cold surroundings. In addition, it is interesting to note that the value of the heat transfer coefficient on the hot side does not influence the value of the critical heat generation.

Region II exists for values of the heat generation greater than the critical value \( \dot{W}_c > \dot{W}_{c,\text{critical}} \) and is referred to as the super-critical region. There is now heat dissipated from both sides of the porous solid slab, so that both the surrounding fluids act as heat sinks. The heat generated within the porous solid slab is distributed depending upon the position of the maximum temperature within the slab and this maximum temperature is greater than that of the hotter surrounding fluid. The position of the maximum temperature is given by Eq. (26), denoted now by \( x_{\text{max}} \), which will lie in the range \(-L \leq x_{\text{max}} \leq L\) in this region II. The maximum temperature is obtained by substituting this value in the expression for the temperature profile Eq. (20). The heat fluxes \( \dot{q} = \dot{Q} / A_s \) at the two surfaces of the slab are as follows:

\[
\dot{q}_{-2} = -U \left\{ 2W_c L \left[ \frac{L}{\lambda} + \frac{1}{\alpha_1} \right] \right\} (T_{-2} - T_{-1})
\]  

(28)

\[
\dot{q}_{-1} = U \left\{ 2W_c L \left[ \frac{L}{\lambda} + \frac{1}{\alpha_2} \right] \right\} (T_{-2} - T_{-1})
\]  

(29)

The total heat flux \( \dot{q}_T \) dissipated from the porous solid slab is obtained by summing the modulii of Eq. (28) and (29), and is the heat generated within the slab

\[
\dot{q}_T = |\dot{q}_{-2}| + |\dot{q}_{-1}| = 2W_c L
\]  

(30)

The heat that is dissipated from both sides of the porous solid slab differs not only due to the position of the maximum temperature, which corresponds to the first term in the brackets of Eq. (28) and (29), but by the heat flow given by Eq. (23) for the situation of no heat generation in the slab. Note that this is the last term in the brackets in Eq. (28) and (29) and it is either added to the first term or removed depending upon the side with the highest temperature.

A typical temperature profile in this region II is plotted in Fig. 2. So unlike the symmetric problem in Section 2.1, the asymmetric problem must first be in the super-critical region before the maximum temperature in the porous solid slab is greater than the hotter surrounding fluid. This is important if there is the possibility that the maximum temperature becomes greater than the melting point of the material or there is the potential of a chemical runaway.

This model for the thermal behaviour of a porous solid with asymmetric heat transfer boundary conditions shall be known as the Heggs-Dare model. Similar results have been obtained for a long hollow cylinder with uniform heat generation, but space precludes inclusion of this model (Dare 2005).

The Semenov model is also applied for asymmetric problems (Babrauskas 2003) and utilises the assumption that \( \lambda \rightarrow \infty \), so that the temperature will be uniform throughout the slab. The temperature of the porous solid slab is obtained from Eq. (20) or more simply may be evaluated by performing a heat balance over the slab to give:

\[
T = \frac{(\alpha_2 T_{-2} + \alpha_1 T_{-1}) + 2L \dot{W}_c}{\alpha_1 + \alpha_2}
\]  

(31)
There will be a critical value of the heat generation that will be the boundary between Regions I and II and this is obtained from Eq. (27) by putting $\lambda = \infty$ to give

$$W_{v,\text{critical}} = \frac{T_{\text{hot}} - T_{\text{cold}}}{2L} \frac{2}{\alpha_{\text{cold}}}$$  \hspace{1cm} (32)$$

A typical temperature profile for the Semenov model in Region I is plotted in Fig. 3.

![Temperature profile for the Semenov model](image)

Figure 3. Temperature profile in Region I for the Semenov model.

The Frank-Kamenetskii assumption that $\alpha \rightarrow \infty$, is also used for asymmetric problems and this implies that the temperatures at the slab sides will be equal to the bulk fluid temperatures. This simplifies the asymmetric boundary conditions:

$$T_{x=-L} = T_{x=2}$$  \hspace{1cm} (33)$$

$$T_{x=L} = T_{x=1}$$  \hspace{1cm} (34)$$

Applying the boundary conditions (33) and (34) to the governing differential equation, Eq. (17), the temperature profile for the Frank-Kamenetskii model is as follows:

$$T = \frac{W_{v}}{2\lambda} L^2 \left( 1 - \frac{x^2}{L^2} \right) \left( \frac{T_{x=1} - T_{x=2}}{2L} \right) + \frac{T_{x=1} + T_{x=2}}{2}$$  \hspace{1cm} (35)$$

The critical rate of heat generation may be evaluated in the same way as for the Semenov model, but this time putting $\alpha_{\text{cold}} = \infty$ into Eq. (27) and this gives

$$W_{v,\text{critical}} = \frac{(T_{\text{hot}} - T_{\text{cold}})}{2L^2}$$  \hspace{1cm} (36)$$

A typical temperature profile across the porous solid slab in region II for the Frank-Kamenetskii model is plotted in Fig. 4. The temperatures at the edges of the slab are now equal to the surrounding fluid temperatures.

### 2.4. Comparison of the Heggs-Dare Model with Semenov and Frank-Kamenetskii for the Asymmetric Infinite Porous Slab with a Constant Uniform Heat Generation

The reciprocal of the expression $W_{v,\text{critical}}$ for the Heggs-Dare model Eq. (27) is equal to the sum of the reciprocals of $W_{v,\text{critical}}$ for the Semenov and Frank-Kamenetskii models, Eq. (32) and (33).

$$\frac{1}{W_{v,\text{critical}}^{HD}} = \frac{1}{W_{v,\text{critical}}^{\text{Semenov}}} + \frac{1}{W_{v,\text{critical}}^{\text{Frank-Kamenetskii}}}$$  \hspace{1cm} (34)$$
Hence the simplified models of Semenov and Frank-Kamenetskii predict values of the critical heat generation term higher than the Heggs-Dare model. The difference in magnitude of these values will depend upon the values of the thermal conductivity of the porous material and the heat transfer coefficient on the cold side of the slab. Furthermore, for values of the heat generation term greater than \( W_{c,\text{critical}} \) for the Heggs-Dare, the maximum temperatures predicted by the Semenov and Frank-Kamenetskii will be lower than the maximum temperature obtained from the Heggs-Dare model. These are illustrated in Fig. 5, where the values of the parameters employed to generate the profiles are listed in Table 1.

The temperature profiles displayed in Fig. 5 have been obtained with a heat generation value equivalent to the critical value of the heat generation for the Semenov model, which for this problem is approximately twenty times greater than the critical values of the two other models. The Heggs-Dare model has the lowest critical value of the heat generation term, which is to be expected from Eq. (34). The temperature for the Semenov model is fixed throughout the porous solid slab at 60°C, whereas the maximum temperature is 105.1°C for the Heggs-Dare model and lower at 100.1°C for the Frank-Kamenetskii model. For the specifications of this problem, the heat flux if \( W = 0 \) is –0.79 W/m² and the total heat flux generated within the porous solid slab is 30.0 W/m². For this particular problem, the hottest surrounding temperature is at \( x=L \) and the heat fluxes dissipated at the surfaces of the porous solid slab are listed in Table 1. The largest heat flux into the cold fluid surroundings occurs for the Semenov model at 30 W/m², because the heat generation term is the critical value. For this model no heat is dissipated into the hot fluid surroundings. For the other two models, the heat flux generated within the porous solid slab leaves from both surfaces and the largest heat fluxes are dissipated into the cold surrounding fluid with the Frank-Kamenetskii value being slightly larger than the Heggs-Dare model. However, the largest value of the heat dissipated into the hot surrounding fluid is obtained from the Heggs-Dare model. This is to be expected for this problem, because both the heat transfer coefficients are same and so from Eq. (28) and (29), the heat flow at the surfaces is half the total heat flux and either plus or minus the heat flux when \( W = 0 \).

Table 1. Values employed to generate the temperature profiles plotted in Fig. 5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Heggs-Dare</th>
<th>Frank-Kamenetskii</th>
<th>Semenov</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half thickness of the slab</td>
<td>( L )</td>
<td>m</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Uniform heating value</td>
<td>( W_1 )</td>
<td>W/m³</td>
<td>12.5</td>
<td>12.5</td>
<td>12.5</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>( \lambda_{\text{K}} )</td>
<td>W/m K</td>
<td>0.2</td>
<td>0.2</td>
<td>∞</td>
</tr>
<tr>
<td>Air temperature 1</td>
<td>( T_{\infty 1} )</td>
<td>°C</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Air temperature 2</td>
<td>( T_{\infty 2} )</td>
<td>°C</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Heat transfer coefficient 1</td>
<td>( \alpha_1 )</td>
<td>W/m² K</td>
<td>3</td>
<td>∞</td>
<td>3</td>
</tr>
<tr>
<td>Heat transfer coefficient 2</td>
<td>( \alpha_2 )</td>
<td>W/m² K</td>
<td>3</td>
<td>∞</td>
<td>3</td>
</tr>
<tr>
<td>Critical heat value</td>
<td>( W_{c,\text{critical}} )</td>
<td>W/m³</td>
<td>0.658</td>
<td>0.694</td>
<td>12.5</td>
</tr>
<tr>
<td>Maximum temperature</td>
<td>( T_{\max} )</td>
<td>°C</td>
<td>105.1</td>
<td>100.1</td>
<td>60.0</td>
</tr>
<tr>
<td>Position of maximum</td>
<td>( x_{\max} )</td>
<td>m</td>
<td>0.063</td>
<td>0.067</td>
<td>Throughout</td>
</tr>
<tr>
<td>Heat flux at ( x=L )</td>
<td>( q_{\infty 2} )</td>
<td>W/m²</td>
<td>14.21</td>
<td>14.17</td>
<td>0</td>
</tr>
<tr>
<td>Heat flux at ( x=-L )</td>
<td>( q_{\infty 1} )</td>
<td>W/m²</td>
<td>–15.79</td>
<td>–15.83</td>
<td>–30.0</td>
</tr>
</tbody>
</table>
If for this problem the heat generation term was temperature dependent, then the Heggs-Dare model should be employed, especially if there is a chance that the melting point of the porous material could be reached or if a thermal runaway could happen. The use of the other two models should be used with caution when the heat generation term is temperature dependent.

2.5. Conclusions

According to the Heggs-Dare model, the temperature profile within the slab consists of the temperature profile due to heat generation within the slab superposed upon the temperature profile due to the heat flow across the slab with no heat generation. Unlike the symmetric problem, where the temperature within the porous material is always greater than that of the surrounding fluids, two temperature regions exist for the asymmetric situation. Region I exists between the temperature profile for the case of no heat generation and the profile obtained at a critical value of the heat generation term $W_{v,\text{critical}}$. This critical rate of heat generation exists where the temperature gradient is equal to zero at the hot solid boundary. Below this critical rate, heat energy flows from the hot fluid to the cold fluid. The rate of heat flow between the hot and cold surrounding fluids decreases as $W_{v,\text{critical}}$ is approached until at $W_{v} = W_{v,\text{critical}}$, heat flux ceases across the hot solid boundary and all the heat generated within the porous solid slab is dissipated into the cold surrounding fluid. Above this critical value is Region II, which is termed the supercritical region and the total generated heat flux is distributed to both fluids, so that both surrounding fluids act as heat sinks. The largest heat flux occurs at the cold side and the heat flux into the hot side is lower than that into the cold side by twice the heat flux corresponding to the situation when there is no heat generation term.

The Semenov and Frank-Kamenetskii models are the two extremes of the Heggs-Dare model for asymmetric heat transfer with constant and uniform heat generation and will predict lower maximum temperatures. These two models will however predict larger values of the critical heat generation term, which will imply falsely that the stable Region I extends to larger values of temperature than the more realistic Heggs-Dare model.

It is imperative that any simulation of an asymmetric problem with a temperature dependent heat generation term must be solved by a model incorporating finite values of the thermal conductivity of the porous material and with finite values of both heat transfer coefficients.

3. References


